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Exp. 05 ⇒ A Reta.

01) $P = A + t(B-A)$

P(x,y,z)
A(2,-3,4)
B(1,-1,2)

I) $(x,y,z) = (2,-3,4) + t(-1,2,-2)$

II) $C(5/2,-4,5)$

$$\begin{cases} x = 2 - t \\ y = -3 + 2t \\ z = 4 - 2t \end{cases}$$

$5/2 = 2 - t$
$t = -1/2$
$-4 = -3 + 2t$
$t = -1/2$
$5 = 4 - 2t$
$t = -1/2$

$C \in r$.

III) $D(-1,3,4)$

$x = 2 - t$ $\quad y = -3 + 2t$ $\quad z = 4 - 2t$
$-1 = 2 - t$ $\quad 3 = -3 + 2t$ $\quad 4 = 4 - 2t$
$t = 3$ $\quad t = 3$ $\quad -2t = 0$
$t = 0$

$D \notin r$

02) $r: (x,y,z) = (-1,2,3) + t(2,-3,0)$

$$r: \begin{cases} x = -1 + 2t \\ y = 2 - 3t \\ z = 3 \end{cases}$$

03) $\vec{v} = (0,0,1)$

$r: (x,y,z) = (1,4,3) + t(0,0,1)$

A(1,2,3)

$$r: \begin{cases} x = 1 \\ y = 2 \\ z = 3 + t \end{cases}$$

04) $$r: \begin{cases} x = 2 + t \\ y = 3 - t \\ z = -4 + 2t \end{cases}$$

a) $y = 6$
P(x,6,z)

$6 = 3 - t$
$t = -3$

$x = 2 + t$
$x = -1$

$z = -4 + 2(-3)$
$z = -10$

$P(-1,6,-10)$

b) $x = y$
Q(x,y,z)

$2 + t = 3 - t$
$2t = 1$
$t = 1/2$

$x = 2 + 1/2 = 5/2$
$y = 3 - 1/2 = 5/2$
$z = -4 + 2(1/2) = -3$

$Q(5/2, 5/2, -3)$

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c) $z = 4x$

$R(x,y,z)$

$-4 + 2t = 4(2 + t)$

$-4 + 2t = 8 + 4t$

$2t = -12$

$t = -6$

$x = 2 - 6 = -4$

$y = 3 - (-6) = 9$

$z = -4 + 2(-6) = -16$

$R(-4, 9, -16)$

05) $A(4,-3,-2)$

$s: (x,y,z) = (1,2,3) + t(3,-4,-1)$

$r: (m,n,-5) = (4,-3,-2) + t(3,-4,-1)$

$$s: \begin{cases} x = 1 + 3t \\ y = 2 - 4t \\ z = 3 - t \end{cases}$$

$-5 = -2 - t$

$t = 3$

$m = 4 + 3t$ $n = -3 - 4t$

$m = 4 + 3 \cdot 3$ $n = -3 - 4 \cdot 3$

$m = 13$ $n = -15$

obs: como $s // r$, os vetores diretos são iguais.

06) a) $P = A + t(B - A)$

$(x,y,z) = (1,-1,2) + t(1,2,-2)$

$$r: \begin{cases} x = 1 + t \\ y = -1 + 2t \\ z = 2 - 2t \end{cases}$$

c) $(x,y,z) = (1,2,3) + t(0,1,-1)$

$$r: \begin{cases} x = 1 \\ y = 2 + t \\ z = 3 - t \end{cases}$$

b) $(x,y,z) = (3,1,4) + t(0,-3,-2)$

$$r: \begin{cases} x = 3 \\ y = 1 - 3t \\ z = 4 - 2t \end{cases}$$

d) $(x,y,z) = (0,0,0) + t(0,1,0)$

$$r: \begin{cases} x = 0 \\ y = t \\ z = 0 \end{cases}$$

07) a) $A(2,0,4)$

$B(0,0,4)$

$P(x,y,z)$

$P = A + t(B - A)$

$(x,y,z) = (2,0,4) + t(-2,0,0)$

$$r: \begin{cases} x = 2 - 2t \\ y = 0 \\ z = 4 \end{cases}$$

b) $C(0,3,0)$; $D(2,3,0)$; $P(x,y,z)$

$P = C + t(D - C)$

$(x,y,z) = (0,3,0) + t(2,0,0)$

$r: \begin{cases} x = 2t \\ y = 3 \\ z = 0 \end{cases}$

c) $A(2,0,4)$; $D(2,3,0)$; $P(x,y,z)$

$P = A + t(D - A)$

$(x,y,z) = (2,0,4) + t(0,3,-4)$

$r: \begin{cases} x = 2 \\ y = 3t \\ z = 4 - 4t \end{cases}$

d) $B(0,0,4)$; $C(0,3,0)$; $P(x,y,z)$

$P = B + t(C - B)$

$(x,y,z) = (0,0,4) + t(0,3,-4)$

$r: \begin{cases} x = 0 \\ y = 3t \\ z = 4 - 4t \end{cases}$

e) $D(2,3,0)$; $E(2,0,0)$; $P(x,y,z)$

$P = D + t(E - D)$

$(x,y,z) = (2,3,0) + t(0,-3,0)$

$r: \begin{cases} x = 2 \\ y = 3 - 3t \\ z = 0 \end{cases}$

f) $B(0,0,4)$; $D(2,3,0)$; $P(x,y,z)$

$P = B + t(D - B)$

$(x,y,z) = (0,0,4) + t(2,3,-4)$

$r: \begin{cases} x = 2t \\ y = 3t \\ z = 4 - 4t \end{cases}$

8) $P(m,1,n)$; $A(3,-1,4)$; $B(4,-3,-1)$

$P = A + t(B - A)$

$(m,1,n) = (3,-1,4) + t(1,-2,-5)$

$r: \begin{cases} m = 3 + t \\ 1 = -1 - 2t \\ n = 4 - 5t \end{cases}$

$1 = -1 - 2t$; $-2t = 2$; $t = -1$

$m = 3 + t$; $m = 3 - 1$; $m = 2$

$n = 4 - 5t$; $n = 4 - 5(-1)$; $n = 4 + 5$; $n = 9$

$P(2,1,9)$

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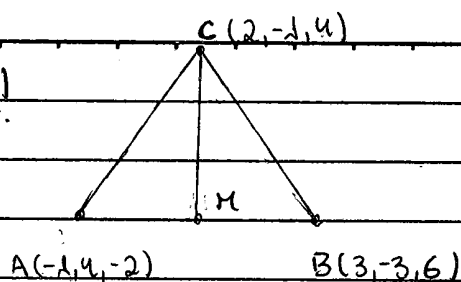
09)

$C(2,-1,4)$, $A(-1,4,-2)$, $B(3,-3,6)$, M

$$M = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2}\right)$$

$$M = \left(\frac{-1+3}{2}, \frac{4+(-3)}{2}, \frac{-2+6}{2}\right)$$

$$M = (1, 1/2, 2)$$

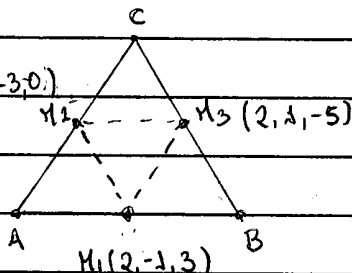
$\vec{MC} = (1, -3/2, 2)$

$P = M + t(C - M)$

$(x,y,z) = (1, 1/2, 2) + t(1, -3/2, -2)$

$$r: \begin{cases} x = 1 + t \\ y = 1/2 - 3/2\,t \\ z = 2 + 2t \end{cases}$$

10) $(1,-3,0)$

C, A, B, M_2, $M_3(2,1,-5)$, $M_1(2,-1,3)$

$P = M_1 + t(M_3 - M_2)$

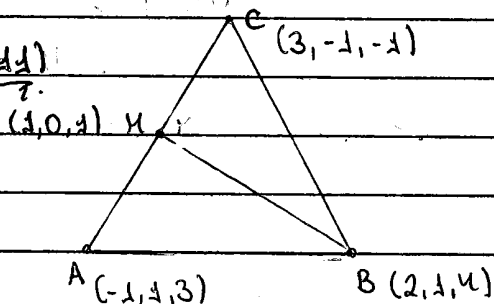
$(x,y,z) = (2,-1,3) + t(1,4,-5)$

$$r: \begin{cases} x = 2 + t \\ y = -1 + 4t \\ z = 3 - 5t \end{cases}$$

$\vec{M_2M_3} = \frac{1}{2}(\vec{AC} + \vec{CB})$

$\vec{M_2M_3} = \frac{1}{2}\vec{AB} \Rightarrow \vec{M_2M_3} = (1,4,-5)$

11)

$C(3,-1,-1)$, $M(1,0,1)$, $A(-1,1,3)$, $B(2,1,4)$

I) $\vec{AB} = \vec{v}$

$P = A + t(B - A)$

$(x,y,z) = (-1,1,3) + t(3,0,1)$

$$r: \begin{cases} x = -1 + 3t \\ y = 1 \\ z = 3 + t \end{cases}$$

II) $\vec{v} = \vec{AC}$ $P = A + t(C - A)$

$(x,y,z) = (-1,1,3) + t(4,-2,-4)$

$$r: \begin{cases} x = -1 + 4t \\ y = 1 - 2t \\ z = 3 - 4t. \end{cases}$$

III) $\vec{v} = \vec{BC}$ $\qquad P = B + t(C - B)$

$P(x,y,z) \qquad (x,y,z) = (2,1,4) + t(1,-2,-5)$

$$r: \begin{cases} x = 2 + t \\ y = 1 - 2t \\ z = 4 - 5t \end{cases}$$

IV) $\vec{MB} = \vec{v}$ $\qquad P = M + t(B - M)$

$P(x,y,z) \qquad (x,y,z) = (1,0,1) + t(-1,1,3)$

$$r: \begin{cases} x = 1 + t \\ y = t \\ z = 1 + 3t \end{cases}$$

12) $r: \dfrac{x-3}{-1} = \dfrac{y+1}{2} = \dfrac{z-2}{-2} \Rightarrow r: (x,y,z) = (3,-1,2) + t(-1,2,-2)$

$$\Rightarrow r: \begin{cases} x = 3 - t \\ y = -1 + 2t \\ z = 2 - 2t \end{cases}$$

I) $P_1(5,-5,6)$

$5 = 3 - t \qquad -5 = -1 + 2t \qquad 6 = 2 - 2t$

$t = -2 \qquad t = -2 \qquad t = -2 \qquad$ $P_1 \in r$

II) $P_2(4,-1,12)$

$4 = 3 - t \qquad -1 = -1 + 2t \qquad 12 = 2 - 2t \qquad$ $P_2 \notin r$.

$t = -1 \qquad t = 0 \qquad t = -5$

13) $r: (x,y,z) = (1,-3,0) + t(2,-1,4)$

$$\Rightarrow r: \begin{cases} x = 1 + 2t \\ y = -3 - t \\ z = 4t \end{cases}$$

a) $x = 5$; $\quad 5 = 1 + 2t \qquad y = -3 - t$

$2t = 4 \qquad y = -3 - 2$

$t = 2 \qquad y = -5$

$z = 4t$

$z = 4 \cdot 2 \qquad$ $P(5,-5,8)$

$z = 8$.

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b) $y=2$;

$y=-3-t$ $\quad$ $x=1+2t$ $\quad$ $z=4t$

$2=-3-t$ $\quad$ $x=1+2(-5)$ $\quad$ $z=4(-5)$

$t=-5$ $\quad$ $x=-9$ $\quad$ $z=-20$

$Q(-9,2,-20)$

14) $P(1,y,z)$ $\quad$ r: $(2x,3y,z)=(-1,2,-4)+t(3,2,1)$
3.

$$r:\begin{cases}2x=-1+3t\\3y=2+2t\\z=-4+t\end{cases}$$

$2=-1+3t$ $\quad$ $3y=2+2t$

$t=1$ $\quad$ $3y=2+2$

$y=4/3$

$z=-4+1$

$z=-3$

$P(1,4/3,-3)$

17) vetor diretor $(x,2,z)$

$$r: \frac{2x+1}{3}=\frac{3y-2}{2}=\frac{z+4}{1} \Rightarrow \frac{x+1/2}{3/2}=\frac{y-2/3}{2/3}=\frac{z+4}{1}$$

r: $(x,y,z)=(-1/2,2/3,-4)+t(3/2,2/3,1)$

↓

vetor diretor.

$3\cdot(3/2,2/3,1)$

$(9/2,2,3)$

Seg | Ter | Qua | Qui | Sex | Sáb | Dom   / /

15) a) $A(4,0,-3)$ $\vec{v}=(2,4,5)$

$r: (x,y,z) = (4,0,-3) + t(2,4,5)$

$\Rightarrow r: \frac{x-4}{2} = \frac{y}{4} = \frac{z+3}{5}$

$\Rightarrow r: \begin{cases} y = 2x-8 \\ z = 5x/2 - 13 \end{cases}$

b) $A(1,-2,3)$
$B(3,-1,-1)$
$P(x,y,z)$

$P = A + t(B-A)$

$r: (x,y,z) = (1,-2,3) + t(2,1,-4)$

$\Rightarrow r: \frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{-4}$

$\frac{x-1}{2} = \frac{y+2}{1}$

$\frac{x-1}{2} = y+2$

$y = \frac{x-1-4}{2} = \frac{x-5}{2}$

$\frac{x-1}{2} = \frac{z-3}{-4}$

$z-3 = -2x+2$

$z = -2x+5$

$\Rightarrow r: \begin{cases} y = \frac{x-5}{2} \\ z = -2x+5 \end{cases}$ //

c) $A(-1,2,3)$
$B(2,-1,3)$
$P(x,y,z)$

$P = A + t(B-A)$

$r: (x,y,z) = (-1,2,3) + t(3,-3,0)$

$\Rightarrow r: \frac{x+1}{3} = \frac{y-2}{-3} = \frac{z-3}{0}$

$\frac{x+1}{3} = \frac{y-2}{-3}$

$y-2 = -x-1$

$y = -x+1$

$\frac{x+1}{3} = \frac{z-3}{0}$

$z-3 = 0$

$z = 3$

$\Rightarrow r: \begin{cases} y = -x+1 \\ z = 3 \end{cases}$ //

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d) $r: \begin{cases} x = 2 - t \\ y = 3t \\ z = 4t - 5 \end{cases}$ $\Rightarrow r: \frac{x-2}{-1} = \frac{y}{3} = \frac{z+5}{4}$

Equações reduzidas:

$\frac{x-2}{-1} = \frac{y}{3}$ $\frac{x-2}{-1} = \frac{z+5}{4}$ $\Rightarrow r: \begin{cases} y = -3x + 6 \\ z = -4x + 3 \end{cases}$

$y = -3x + 6$ $z + 5 = -4x + 8$

$z = -4x + 3$

16) A(-1,6,3) | P = A + t(B - A)

B(2,2,1) | r: (x,y,z) = (-1,6,3) + t(3,-4,-2)

P(x,y,z) |

$\frac{x+1}{3} = \frac{z-3}{-2}$ $\frac{y-6}{-4} = \frac{z-3}{-2}$

Equações simétricas:

$\Rightarrow r: \frac{x+1}{3} = \frac{y-6}{-4} = \frac{z-3}{-2}$ $x + 1 = \frac{-3z+9}{2}$ $y - 6 = 2z - 6$

$y = 2z$

$x = \frac{-3z+7}{2}$

Equações reduzidas:

$\Rightarrow r: \begin{cases} x = -\frac{3z}{2} + \frac{7}{2} \\ y = 2z \end{cases}$

17) $r: \begin{cases} y = 2x + 3 \\ z = x - 1 \end{cases}$ a) $y = 9$; $9 = 2x + 3$ $z = 3 - 1$ P(3,9,2)

$2x = 6$ $z = 2$

$x = 3$

b) $x = 2z$; $z = x - 1$ $1 = x - 1$ $y = 2 \cdot 2 + 3$ Q(2,7,1)

$z = 2z - 1$ $x = 2$ $y = 7$

$z = 1$

Seg Ter Qua Qui Sex Sáb Dom / /

e) $y = 3z$; $\begin{cases} 3z = 2x + 3 \\ z = x - 1 \end{cases}$ $3(x-1) = 2x + 3$ $3x - 3 = 2x + 3$ $x = 6$ $z = 6 - 1$ $z = 5$

$y = 3 \cdot 5 = 15$

$R(6, 15, 5)$

18) a) $r: \begin{cases} x = 1 - t \\ y = -1 + 2t \\ z = 2 + t \end{cases}$ $\Rightarrow r: (x, y, z) = (1, -1, 2) + t(-1, 2, 1)$

? ?

?

19) a) $\vec{v} = (1, 0, 0)$ $r: (x, y, z) = (3, -2, 4) + t(1, 0, 0)$

$A(3, -2, 4)$

$P(x, y, z)$ $\Rightarrow r: \begin{cases} x = 3 + t \\ y = -2 \\ z = 4 \end{cases}$

b) $\vec{v} = (0, 1, 0)$ $s: (x, y, z) = (2, 2, 4) + t(0, 1, 0)$

$A(2, 2, 4)$ $\Rightarrow s: \begin{cases} x = 2 \\ y = 2 + t \\ z = 4 \end{cases}$

c) $\vec{v}(0, 0, 1)$ $p: (x, y, z) = (-2, 3, 4) + t(0, 0, 1)$

$A(-2, 3, 4)$ $\Rightarrow p: \begin{cases} x = -2 \\ y = 3 \\ z = 4 + t \end{cases}$

d) $\vec{v}(3, -2, 0)$ $r: (x, y, z) = (4, -1, 3) + t(3, -2, 0)$

$A(4, -1, 3)$ $\Rightarrow r: \begin{cases} x = 4 + 3t \\ y = -1 - 2t \\ z = 3 \end{cases}$

e) $A(3,-1,3)$ $\quad P = A + t(B-A)$

$B(3,3,4)$ $\quad r: (x,y,z) = (3,-1,3) + t(0,4,1)$

$P(x,y,z)$

$$\Rightarrow r: \begin{cases} x = 3 \\ y = -1 + 4t \\ z = 3 + t \end{cases}$$

20) 3. $A(4,-5,3)$ $\quad$ I) $Ox \Rightarrow \vec{v} = (1,0,0)$

$r: (x,y,z) = (4,-5,3) + t(1,0,0)$

$$\Rightarrow r: \begin{cases} x = 4 + t \\ y = -5 \\ z = 3 \end{cases}$$

II) $Oy \Rightarrow \vec{v} = (0,1,0)$

$s: (x,y,z) = (4,-5,3) + t(0,1,0)$

$$\Rightarrow s: \begin{cases} x = 4 \\ y = -5 + t \\ z = 3 \end{cases}$$

III) $Oz \Rightarrow \vec{v} = (0,0,1)$

$p: (x,y,z) = (4,-5,3) + t(0,0,1)$

$$\Rightarrow p: \begin{cases} x = 4 \\ y = -5 \\ z = 3 + t \end{cases}$$

21) 3. a)

$$r_1: \begin{cases} x = -2 - t \\ y = t \\ z = 3 - 2t \end{cases} \qquad r_2: \begin{cases} x = 2t \\ y = -6 + t \\ z = 1 + t \end{cases}$$

$\vec{v}_1 = (-1, 1, -2)$

$\vec{v}_2 = (2, 1, 1)$

$$\cos\theta = \frac{|\vec{v}_1 \cdot \vec{v}_2|}{|\vec{v}_1| \cdot |\vec{v}_2|}$$

$|\vec{v}_1| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$

$|\vec{v}_2| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

$$\cos\theta = \frac{|(-1,1,-2) \cdot (2,1,1)|}{\sqrt{6} \cdot \sqrt{6}}$$

$$\cos\theta = \frac{|-2 + 1 - 2|}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2} = 60°$$

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b) $r_1: \begin{cases} y = -2x+3 \\ z = x-1 \end{cases}$ $r_2: y = \frac{z+1}{-1};\ x = 4$ $\vec{v_2}(1,-1,0)$
$\vec{v_1}(1,-2,1)$

$x=0;\ A(0,3,-2)$
$x=1;\ B(1,1,-1)$
$\vec{v_1} = (1,-2,1)$

$\cos\theta = \frac{|\vec{v_1}\cdot\vec{v_2}|}{|\vec{v_1}|\cdot|\vec{v_2}|} = \frac{|(1,-2,1)\cdot(1,-1,0)|}{\sqrt{6}\cdot\sqrt{2}}$

$= \frac{1+2}{\sqrt{12}} = \frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} = 30°$

$|\vec{v_1}| = \sqrt{1^2+(-2)^2+1^2} = \sqrt{6}$
$|\vec{v_2}| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

c) $r_1: \begin{cases} x = 1+\sqrt{2}t \\ y = t \\ z = 5-3t \end{cases}$ $r_2: \begin{cases} x = 3 \\ y = 2 \end{cases}$ $\vec{v_1} = (\sqrt{2},1,-3)$
$\vec{v_2} = (0,0,1)$

$|\vec{v_1}| = \sqrt{\sqrt{2}^2+1^2+(-3)^2} = \sqrt{12}$
$|\vec{v_2}| = \sqrt{1^2} = 1$

$\cos\theta = \frac{|(\sqrt{2},1,-3)\cdot(0,0,1)|}{\sqrt{12}}$

$\cos\theta = \frac{|-3|}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} = 30°$

d) $r_1: \frac{x-4}{2} = \frac{y}{-1} = \frac{z+1}{-2}$ $r_2: \begin{cases} x = 1 \\ \frac{y}{4} = \frac{z-2}{3} \end{cases}$ $\vec{v_1} = (2,-1,-2)$
$\vec{v_2} = (0,4,3)$

$|\vec{v_1}| = \sqrt{2^2+(-1)^2+(-2)^2} = \sqrt{9} = 3$
$|\vec{v_2}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$\cos\theta = \frac{|(2,-1,-2)\cdot(0,4,3)|}{3\cdot5}$

$\cos\theta = \frac{|-10|}{3\cdot5} = \frac{10}{3\cdot5} = \frac{2}{3} \approx 48{,}11°$

/ /

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22) a) $r_1: \frac{x-2}{4} = \frac{y}{5} = \frac{z}{3}$ $\quad r_2: \begin{cases} y = nx+5 \\ z = 2x-2 \end{cases}$ $\quad x=0;\ A(0,5,-2)$ $\quad x=1;\ B(1,n+5,0)$

$\vec{v_1} = (4,5,3)$

$\vec{v_2} = (1,n,2)$

$\cos\theta = 30°$

$|\vec{v_1}| = \sqrt{4^2+5^2+(3)^2} = \sqrt{50} = 5\sqrt{2}$

$|\vec{v_2}| = \sqrt{1^2+n^2+2^2} = \sqrt{n^2+5}$

$$\frac{|(4,5,3)\cdot(1,n,2)|}{5\sqrt{2}\cdot\sqrt{n^2+5}} = \frac{\sqrt{3}}{2}$$

$$\frac{10n+20}{5\cdot\sqrt{2n^2+10}} = \sqrt{3}$$

$$10n+20 = 5\cdot\sqrt{6n^2+30}$$

$$(10n+20)^2 = 5^2\cdot(\sqrt{6n^2+30})^2$$

$$100n^2+400n+400 = 25(6n^2+30)$$

$$100n^2+400n+400 = 150n^2+750$$

$$50n^2-400n+350=0$$

$$5n^2-40n+35=0$$

$$n^2-8n+7=0$$

$$\Delta = 64-28 = 36$$

$$n = \frac{8\pm6}{2}$$

$n' = 7$

$n'' = 1$

b) $r_1: \begin{cases} y = nx-1 \\ z = 2x \end{cases}$ $\quad r_2$: eixo Oy $\quad |\vec{v_1}| = 1$

$\vec{v_1} = (0,1,0)$ $\quad |\vec{v_2}| = \sqrt{n^2+5}$

$x=0;\ A(0,-1,0)$

$x=1;\ B(1,n-1,2)$ $\quad \cos\theta = 30°$

$\vec{v_2} = (1,n,2)$

$$\frac{|(0,1,0)\cdot(1,n,2)|}{\sqrt{n^2+5}} = \frac{\sqrt{3}}{2}$$

$$n\cdot 2 = \sqrt{3n^2+15}$$

$$(2n)^2 = (\sqrt{3n^2+15})^2$$

$$4n^2 = 3n^2+15$$

$$n^2 = 15$$

$$n = \pm\sqrt{15}$$

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23) a)

$r_1: \begin{cases} x = 2mt - 3 \\ y = 1 + 3t \\ z = -4t \end{cases}$

$r_2: \begin{cases} x = 2y - 1 \\ z = -y + 4 \end{cases}$

$r_1 \perp r_2$

$\vec{v}_1 \cdot \vec{v}_2 = 0$

$\vec{v}_1 = (2m, 3, -4)$

$y = 0;\ A(-1, 0, 4)$

$y = 1;\ B(1, 1, 3)$

$\vec{v}_2 = (2, 1, -1)$

$\vec{v}_1 \cdot \vec{v}_2 = 0$

$(2m, 3, -4) \cdot (2, 1, -1) = 0$

$4m + 3 + 4 = 0$

$4m = -7$

$m = -7/4$

b) $r_1: \begin{cases} y = mx + 3 \\ z = x - 1 \end{cases}$

r_2: reta por $A(1, 0, m)$ e $B(-2, 2m, 2m)$

$r_2: (x, y, z) = (1, 0, m) + t(-3, 2m, m)$

$\vec{v}_2 = (-3, 2m, m)$

$x = 0;\ A(0, 3, -1)$

$x = 1;\ B(1, m+3, 0)$

$\vec{v}_1 = (1, m, 1)$

$r_1 \perp r_2$

$\vec{v}_1 \cdot \vec{v}_2 = 0$

$(1, m, 1) \cdot (-3, 2m, m) = 0$

$-3 + 2m^2 + m = 0$

$\Delta = 1 + 24 = 25$

$m = \frac{-1 \pm 5}{4}$

$m' = -6/4 = -3/2$

$m'' = 1$

24) a) $A(3, 2, -1)$

$r_1: \begin{cases} x = 3 \\ y = -1 \end{cases}$

$r_2: \begin{cases} y = x - 3 \\ z = -2x + 3 \end{cases}$

$\vec{v}_1 = (0, 0, 1)$

$x = 0;\ C(0, -3, 3)$

$x = 1;\ D(1, -2, 1)$

$\vec{v}_2 = (1, 1, -2)$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 1 & -2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 0 \\ 1 & 1 \end{matrix} = 1\hat{j} - 1\hat{i} \Rightarrow \vec{v} = (-1, 1, 0)$$

Equações Paramétricas
que passa pelo ponto A.

$r: (x,y,z) = (3,2,-1) + t(-1,1,0)$

$$\begin{cases} x = 3 - t \\ y = 2 + t \\ z = -1 \end{cases}$$

b) $A(0,0,0)$

$r_1: \dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z-3}{2}$

$\vec{v}_1 = (2,1,2)$

$r_2: \begin{cases} x = 3t \\ y = -t + 1 \\ z = 2 \end{cases}$

$\vec{v}_2 = (3,-1,0)$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & -1 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & 1 \\ 3 & -1 \end{matrix} = 6\hat{j} - 2\hat{k} - 3\hat{k} + 2\hat{i} = 2\hat{i} + 6\hat{j} - 5\hat{k}$$

$\Rightarrow \vec{v} = (2, 6, -5)$

Equações Paramétricas
que passa pelo ponto A.

$r: (x,y,z) = (0,0,0) + t(2,6,-5)$

$$\Rightarrow r \begin{cases} x = 2t \\ y = 6t \\ z = -5t \end{cases}$$

Seg Ter Qua Qui Sex Sáb Dom   / /

c) $r_1: \begin{cases} x = 2 + t_1 \\ y = -1 + 2t_1 \\ z = 3t_1 \end{cases}$ $r_2: \begin{cases} x = 1 - t_2 \\ y = t_2 \\ z = 2 + 2t_2 \end{cases}$

$\vec{v_1} = (1, 2, 3)$ $\vec{v_2} = (-1, 1, 2)$

I) $y = y$

$t_2 = -1 + 2t_1$

II) $x = x$

$2 + t_1 = 1 - t_2$

$2 + t_1 = 1 + 1 - 2t_1$

$3t_1 = 2 - 2$

$t_1 = 0$

$t_2 = -1$

$\vec{v_1} \times \vec{v_2} = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \end{vmatrix} \begin{matrix} 1 & 2 \\ -1 & 1 \end{matrix} = 4-3, -3-2, 1+2$

$\vec{v} = (1, -5, 3)$

Ponto de interseção:

$r_1: \begin{cases} x = 2 \\ y = -1 \\ z = 0 \end{cases}$ $r_2: \begin{cases} x = 2 \\ y = -1 \\ z = 0 \end{cases}$ $I(2, -1, 0)$

$\vec{v} = (1, -5, 3)$

Eq. Paramétrica:

$\begin{cases} x = 2 + t \\ y = -1 - 5t \\ z = 3t \end{cases}$

25) a) $r_1: \begin{cases} y = 2x - 3 \\ z = -x + 5 \end{cases}$ $r_2: \begin{cases} y = -3x + 7 \\ z = x + 1 \end{cases}$

$\Rightarrow r_1: \begin{cases} x = t \\ y = -3 + 2t \\ z = 5 - t \end{cases}$ $\Rightarrow r_2: \begin{cases} x = t \\ y = 7 - 3t \\ z = 1 + t \end{cases}$

$-3 + 2t = 7 - 3t$ $5 - t = t + 1$ $t = t$ $I(2, 1, 3)$

$5t = 10$ $2t = 4$ $2 = 2$

$t = 2$ $t = 2$

b) $r_1: \begin{cases} 3 + 2t_1 \\ -1 - 3t_1 \\ 2 + 4t_1 \end{cases}$ $r_2: \begin{cases} -1 + t_2 \\ 4 - t_2 \\ -8 + 3t_2 \end{cases}$

I) $3 + 2t_1 = -1 + t_2$

$t_2 = 2t_1 + 4$

$t_2 = 2$

II) $-1 - 3t_1 = 4 - 2t_1 - 4$

$-1 = t_1$

$I(1, 2, -2)$

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c) $r_1: \begin{cases} y = 2x - 3 \\ z = -x - 10 \end{cases}$ $\quad r_2: x = \frac{y-4}{3} = \frac{z+1}{-2}$

$\Rightarrow r_1: \begin{cases} x = t \\ y = -3 + 2t \\ z = -10 - t \end{cases}$ $\quad \Rightarrow r_2: \begin{cases} x = t \\ y = 4 + 3t \\ z = -1 - 2t \end{cases}$

I) $t = t$ $\quad$ II) $-3 + 2t = 4 + 3t$ $\quad$ III) $-10 - t = -1 - 2t$

$t = -7$ $\quad$ $t = -9$

Não são concorrentes.

d) $r_1: \begin{cases} x = 2 - t \\ y = 3 - 5t \\ z = 6 - 6t \end{cases}$ $\quad r_2: \begin{cases} -3 + 6h \\ y = 1 + 7h \\ z = -1 + 13h \end{cases}$ $\quad$ I) $2 - t = -3 + 6h$ II) $3 - 5t = 1 + 7h$ III) $6 - 6t = -1 + 13h$

I) $t = 5 - 6h$ $\quad$ II) $3 - 25 + 30h = 1 + 7h$ $\quad$ III) $6 - 30 + 36h = -1 + 13h$

$23h = 23$ $\quad$ $23h = 23$

$t = -1$ $\quad$ $h = 1$ $\quad$ $h = 1$

$I(3, 8, 12)$

e) $r_1: \begin{cases} x = 2 + t \\ y = 4 - 2t \\ z = 1 + 3t \end{cases}$ $\quad r_2: \begin{cases} x = -1 + 4t \\ y = 2 + 3t \\ z = 5 - 2t \end{cases}$ $\quad$ I) $2 + t = -1 + 4t$

$3t = 3$

$t = 1$

II) $4 - 2t = 2 + 3t$ $\quad$ III) $1 + 3t = 5 - 2t$ $\quad$ Não são concorrentes.

$5t = 2$ $\quad$ $5t = 4$

$t = 2/5$ $\quad$ $t = 4/5$

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p) $r_1: \begin{cases} x = 2+t \\ y = 4-t \\ z = -t \end{cases}$ $r_2: \begin{cases} y = 6-x \\ z = 2-x \end{cases}$ $r_2: \begin{cases} x = t \\ y = 6-t \\ z = 2-t \end{cases}$

I) $2+t = t$ II) $4-t = 6-t$ III) $-t = 2-t$

Não são concorrentes.

26) a) $r_1: \begin{cases} x = t_1 \\ y = -5+2t_1 \\ z = 2-t_1 \end{cases}$ $r_2: \begin{cases} x = 5+t_2 \\ y = mt_2 \\ z = -1+t_2 \end{cases}$

I) $t_1 = 5+t_2$

II) $2 - 5 - t_2 = -1 + t_2$

$2t_2 = -2$

$t_2 = -1$

$t_1 = 4$

III) $mt_2 = -5+2t_1$

$-m = -5+8$

$-m = 3$

$m = -3$

b) $r_1: \begin{cases} x = m - t_1 \\ y = 1+t_1 \\ z = 2t_1 \end{cases}$ $\begin{cases} x = 1+3t_2 \\ y = -2+t_2 \\ z = -2t_2 \end{cases}$

I) $2t_1 = -2t_2$

$t_1 = -t_2$

II) $1+t_1 = -2+t_2$

$1-t_2 = -2+t_2$

$2t_2 = 3$

$t_2 = 3/2$

$t_1 = -3/2$

III) $m + 3/2 = 1 + 9/2$

$m = 1 + 6/2$

$m = 8/2$

$m = 4$

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27) 3.

$r_1: \begin{cases} x = 1 + 2t_1 \\ y = -t_1 \\ z = 3 \end{cases}$ $r_2: \begin{cases} x = t_2 \\ y = -1 + t_2 \\ z = 2 + t_2 \end{cases}$

I) $-t_1 = -1 + t_2$

$t_1 = 1 - t_2$

II) $3 = 2 + t_2$

$t_2 = 1$

III) $t_1 = 0$

$I(1, 0, 3)$

$A(0, 1, 0)$

IV) $P = A + t(I - A)$

$(x, y, z) = (0, 1, 0) + t(1, -1, 3)$

$r: \begin{cases} x = t \\ y = 1 - t \\ z = 3t \end{cases}$

$x = \frac{y - 1}{-1} = \frac{z}{3}$

$\begin{cases} y = -x + 1 \\ z = 3x \end{cases}$

28) 3.

$r: \begin{cases} x = 2 + t \\ y = t \\ z = -1 + 2t \end{cases}$

A (2, -1, 2) — B (1, 0, -1); d, d; C (2+t, t, -1+2t)

I) $\vec{AC} = C - A = (t, t+1, 2t+1)$

$\vec{BC} = C - B = (t+1, t, 2t)$

II) $|\vec{AC}| = |\vec{BC}|$

$\sqrt{t^2 + (t+1)^2 + (2t+1)^2} = \sqrt{(t+1)^2 + t^2 + (2t)^2}$

$\cancel{t^2} + \cancel{t^2} + \cancel{2t} + \cancel{1} + \cancel{4t^2} + 4t + 1 = \cancel{t^2} + \cancel{2t} + \cancel{1} + \cancel{t^2} + \cancel{4t^2}$

$4t = -1$

$t = -1/4$

III) $x = 2 + (-1/4) = 3/4$

$y = -1/4$

$z = -1 - 2/4 = -3/2$

$C(3/4, -1/4, -3/2)$

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29) $r: \begin{cases} x = 2 + t \\ y = 1 + 2t \\ z = 3 + 2t \end{cases}$

a) $\vec{AP} = P - A = (t, 2t, 2t)$

A(2,1,3)

I) $|\vec{AP}| = 6$

$\sqrt{t^2 + (2t)^2 + (2t)^2} = 6$

$t^2 + 4t^2 + 4t^2 = 6^2$

$t^2 = 36/9 \Rightarrow t = \pm 2$

II) P(4,5,7) ou

P(0,-3,-1)

b) $\vec{BP} = P - B = (1+t, 2+2t, 2t)$

B(1,-1,3)

I) $|\vec{BP}| = 2$

$\sqrt{(1+t^2) + (2+2t)^2 + (2t)^2} = 2$ $\Rightarrow t = \frac{-10 \pm 8}{18}$

$1 + 2t + t^2 + 4 + 8t + 4t^2 + 4t^2 = 4$

$9t^2 + 10t + 1 = 0$ $t' = -1$

$\Delta = 100 - 36$ $t'' = 1/9$

$\Delta = 64$

II) P(1,-1,1) ou P(17/9, 7/9, 25/9)

30) A(1,3,5) $r: (x,y,z) = (1,3,5) + t(-1,-3,0)$

P(0,0,5) $\Rightarrow r: \frac{x-1}{-1} = \frac{y-3}{-3}; z = 5$

$\vec{AP} = P - A = (-1,-3,0)$

I) $-3x + 3 = -y + 3$ $\Rightarrow r: \begin{cases} y = 3x \\ z = 5 \end{cases}$

$y = 3x$

31) a) A(4,-2,2) $r: x = 2y = -2z \Rightarrow r: \frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$

$s: \frac{x-4}{2} = \frac{y+2}{1} = \frac{z-2}{-1}$ $\vec{v} = (2,1,-1)$

$\Rightarrow s: \begin{cases} x = -2z + 8 \\ y = -z \end{cases}$

Seg Ter Qua Qui Sex Sáb Dom

b) P(0,0,0) I) $r: \frac{2x-1}{3} = \frac{y+2}{-2} = 2z-2$

$r: \frac{x-1/2}{3} = \frac{y+2}{-4} = z-1$ $\vec{v_r} = (3,-4,1)$

II) $s: x = -y = -z$

$\vec{v_s} = (1,-1,-1)$

III) $\vec{v} = \vec{v_r} \times \vec{v_s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 1 & -1 & -1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 3 & -4 \\ 1 & -1 \end{matrix}$

$4\hat{i} + \hat{j} - 3\hat{k} + 4\hat{k} + \hat{i} + 3\hat{j}$

$5\hat{i} + 4\hat{j} + \hat{k}$

$(5,4,1)$

IV) $q: \begin{cases} 5t \\ 4t \\ t \end{cases}$

32) $\vec{v} = \vec{AB} = B - A = (-2,4,-2)$

$\vec{P} = (-2,4,0)$

$\cos\theta = \frac{|\vec{v}\cdot\vec{P}|}{|\vec{v}|\cdot|\vec{P}|} = \frac{|4+16|}{\sqrt{24}\cdot\sqrt{20}} = \frac{20}{4\sqrt{30}} = \frac{5}{\sqrt{30}}$

$\cos\theta = \frac{5\sqrt{30}}{30} = \frac{\sqrt{30}}{6} \rightarrow \theta \cong 24,1°$

33) $r \begin{cases} x = t \\ y = 5t - 7 \\ z = -2t + 6 \end{cases}$ $\vec{v} = (1,5,-2)$ $\vec{P} = (1,5,0)$

I) $z = 0$
$-2x + 6 = 0$
$x = 3$

II) $y = 5x - 7$
$y = 15 - 7$
$y = 8$

III) $s: \begin{cases} x = 3 + t \\ y = 8 + 5t \\ z = 0 \end{cases}$

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34) a) $A(3,4,-2)$

$P(1+t, 2-t, 4+2t)$

$\vec{v_p} = (1,-1,2)$

$\vec{v_p} \cdot \vec{PA} = 0$

$PA(4,0,-2)$

$(1,-1,2) \cdot (2-t, 2+t, -2t-6) = 0$

$2-t+(-2-t)+(-4t-12)=0$

$-6t = 12$

$t = -2$

$s: \begin{cases} x = 3+4h \\ y = 4 \\ z = -2-2h \end{cases}$

b) $|\vec{AP}| = \sqrt{4^2+(-2)^2} = \sqrt{20}$

c) ?